MULTIGROUP METHOD OF THE SOLUTION OF RADIATION TRANSFER

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Abstract: This paper deals with the method of spherical harmonics (P1-approximation) as the way that is used to solve the equation of transfer radiation energy in arc plasma. To calculate the frequency variable in the equation of transfer the multigroup method is supposed to be used. Based on the combination of these two methods the partial differential equations as Bessel modified ones are solved. It enables to obtain an approximate solution of required accuracy.

Keywords: arc plasma, radiation flux, spectral density, spherical harmonics, multigroup method.

1. INTRODUCTION

The equation of transfer radiation energy is considered to be complicated; in general the spectral intensity, which is the dependent variable of this equation, depends on the independent variables ($\vec{r}$, ν, $\vec{\Omega}$). One has to approximate the equation of transfer, either analytically or numerically, in order to obtain a solution. The method of spherical harmonics (PN-approximation) which is based on the transformation the equation of transfer into a set of simultaneous partial differential equations enables to obtain an approximate solution of required accuracy.

2. MULTIGROUP P1-APPROXIMATION

In P1-approximation the angular dependence of the specific intensity is supposed to be represented by the first two terms in a spherical harmonic expansion

$$I_\nu(\vec{r},\nu,\vec{\Omega}) = \varphi_1(\vec{r},\nu) + 3\vec{\varphi}_2(\vec{r},\nu)\cdot\vec{\Omega} \qquad (1)$$

where φ_1 and $\vec{\varphi}_2$ correspond to the density of the radiation field multiplied by velocity of light *c*, and to the radiation flux.

The spectral density of the radiation field is

$$U_\nu(\vec{r},\nu) = \frac{4\pi}{c}\varphi_1(\vec{r},\nu)\,. \qquad (2)$$

Similarly, the expression for radiation flux is

$$\vec{W}_\nu(\vec{r},\nu) = 4\pi\,\vec{\varphi}_2(\vec{r},\nu)\,. \qquad (3)$$

Therefore, we may rewrite equation (1) in terms of radiation field density and radiation flux

$$I_\nu(\vec{r},\nu,\vec{\Omega}) = \frac{c}{4\pi}U_\nu(\vec{r},\nu) + \frac{3}{4\pi}\vec{W}_\nu(\vec{r},\nu)\cdot\vec{\Omega}\,. \qquad (4)$$

One of the methods to calculate the frequency variable in the equation of transfer is the multigroup method [1], which leads to its discretization. One assigns a given photon to one of G frequency groups and all photons within a given group are treated the same from the point of absorption properties of the medium; the absorption coefficient for given frequency group n is supposed to be constant with certain average value

$$k_\nu(\vec{r},\nu,T) = k_n(\vec{r},T), \quad \nu_n \le \nu \le \nu_{n+1}, \quad n = 1,\dots,G. \tag{5}$$

For P1-approximation the multigroup equations of transfer have the form

$$\operatorname{div} \vec{W}_n(\vec{r}) + \bar{k}_n c U_n(\vec{r}) = \bar{k}_n 4\pi \int_{\nu_{n-1}}^{\nu_n} B_\nu d\nu, \tag{6}$$

$$\frac{1}{3} c \cdot \operatorname{grad} U_n(\vec{r}) + \bar{k}_n \vec{W}_n(\vec{r}) = 0. \tag{7}$$

From the equation (7):

$$\vec{W}_n(\vec{r}) = -\frac{1}{3\bar{k}_n} c \operatorname{grad} U_n(\vec{r}). \tag{8}$$

To obtain the elliptical partial differential equation one inserts the previous expression into the equation (6):

$$\operatorname{div}\left(-\frac{1}{3\bar{k}_n} c \operatorname{grad} U_n(\vec{r})\right) + \bar{k}_n c U_n(\vec{r}) = \bar{k}_n 4\pi \int_{\nu_{n-1}}^{\nu_n} B_\nu d\nu. \tag{9}$$

Let's rewrite $B_n(\vec{r}) = \int_{\nu_{n-1}}^{\nu_n} B_\nu d\nu$, $cU_n(\vec{r}) = u_n(\vec{r})$.

The radiation flux is expressed

$$W_n(r) = -\frac{1}{3\bar{k}_n} \frac{\partial u_n(r)}{\partial r}. \tag{10}$$

In case of cylindrically symmetrical isothermal plasma this equation depends only on one variable – radial range r

$$\operatorname{div}\left(-\frac{1}{3\bar{k}_n} \operatorname{grad} u_n(r)\right) + \bar{k}_n u_n(r) = \bar{k}_n 4\pi B_n(r). \tag{11}$$

Let’s express div grad u in cylindrical coordinate system

$$\frac{1}{r}\frac{\partial}{\partial r}\left(-\frac{1}{3\bar{k}_n} r \frac{\partial u_n(r)}{\partial r}\right) + \bar{k}_n u_n(r) = \bar{k}_n 4\pi B_n(r). \tag{12}$$

Due to the fact that functions $\bar{k}_n$ a B_n are dependent on the arc temperature and it decreases from the axis to the edge they are usually regarded as radial range functions. If we assume (to simplify the task) that plasma is isothermal, i.e. the arc temperature remains constant, functions $\bar{k}_n$ a B_n also remain constant for the current solution of the equation (11), resp. (12).

Let’s modify the equation (12)

$$r^2 \frac{\partial^2 u_n(r)}{\partial r^2} + r \frac{\partial u_n(r)}{\partial r} - 3\bar{k}_n^2 r^2 u_n(r) = -3\bar{k}_n^2 r^2 4\pi B_n(r) . \quad (13)$$

Let's solve the equation as Bessel inhomogeneous modified one and taking into account that the left part equals 0.

$$r^2 \frac{\partial^2 u_n(r)}{\partial r^2} + r \frac{\partial u_n(r)}{\partial r} - 3\bar{k}_n^2 r^2 u_n(r) = 0 . \quad (14)$$

The solution of the homogeneous equation (14) is

$$u_n(r) = C_1 I_0\left(\sqrt{3}\bar{k}_n r\right) + C_2 K_0\left(\sqrt{3}\bar{k}_n r\right). \quad (15)$$

The solution of the inhomogeneous equation consists of the solution of the homogeneous equation and u_p, where u_p is the arbitrary particular solution of the inhomogeneous equation. According to the right side of the equation let's find out the solution in the form

$$u_p = Ar^2 + Br + C . \quad (16)$$

After one inserts the previous expression into the equation (16) it's obtained

$$u_p = 4\pi B_n(r) . \quad (17)$$

The resulting solution of the equation (11) is

$$u_n(r) = C_1 I_0\left(\sqrt{3}\,\bar{k}_n\, r\right) + C_2 K_0\left(\sqrt{3}\,\bar{k}_n\, r\right) + 4\pi B_n(r) . \quad (18)$$

Constants C_1 and C_2 are defined according to the boundary conditions of arc axis *r*=0 and arc radius *r*=*R*.

Due to the symmetry of the process the radiation flux $W_n(0)$ equals zero if *r*=0, i.e. $\mathrm{div} W_n(0) = 0$ and the boundary condition could be written as

$$\left. \frac{\partial u_n(r)}{\partial r} \right|_{r=0} = 0 \quad (19)$$

and therefore $C_2 = 0$.

If *r*=*R* the external radiation isn't supposed to enter the plasma so flux parts along the internal normal to field boundary is expressed as

$$\frac{1}{3\bar{k}_n} \left. \frac{\partial u_n(r)}{\partial r} \right|_{r=R} = -\frac{u_n(R)}{2} . \quad (20)$$

Then $C_1 = -\dfrac{4\pi\sqrt{3}\, B_n(r)}{2\, I_1\left(\sqrt{3}\,\bar{k}_n\, R\right) + \sqrt{3}\, I_0\left(\sqrt{3}\,\bar{k}_n\, R\right)}$. (21)

The solution that satisfies the boundary conditions is

$$u_n(r) = -\frac{4\pi\sqrt{3}\, B_n(r)}{2\, I_1\left(\sqrt{3}\,\bar{k}_n\, R\right) + \sqrt{3}\, I_0\left(\sqrt{3}\,\bar{k}_n\, R\right)} I_0\left(\sqrt{3}\,\bar{k}_n\, r\right) + 4\pi B_n(r) . \quad (22)$$

Required flux divergence is

$$\operatorname{div}\vec{W}_n(r) = 4\pi\,\bar{k}_n\, B_n(r) - \bar{k}_n\, u_n(r) = \frac{4\pi\sqrt{3}\,\bar{k}_n\, B_n(r)}{2\, I_1\left(\sqrt{3}\,\bar{k}_n\, R\right) + \sqrt{3}\, I_0\left(\sqrt{3}\bar{k}_n\, R\right)}\, I_0\left(\sqrt{3}\,\bar{k}_n\, r\right). \tag{23}$$

To calculate the mean divergence of the whole cross section one is supposed to sum up the parts of the cyclic cross sections of the plasmatic cylinder on the whole surface and to divide into the same one

$$\left(w_{avg}\right)_n = \frac{2\pi}{\pi R^2}\int r\,\operatorname{div}\vec{W}_n(r)\,d\,r = \frac{2}{R}\,\frac{4\pi\, B_n(r)}{2\, I_1\left(\sqrt{3}\,\bar{k}_n\, R\right) + \sqrt{3}\, I_0\left(\sqrt{3}\,\bar{k}_n R\right)}\, I_1\left(\sqrt{3}\bar{k}_n\, R\right). \tag{24}$$

Summing over all frequency groups gives the net emission

$$\operatorname{div}\vec{W}_R = w_{avg} = \sum_n \left(w_{avg}\right)_n\ . \tag{25}$$

3. RESULTS

Equations (24), (25) has been solved for isothermal air plasma cylinder at the pressure 1 atm, in temperature range (10 000 – 30 000 K), and for various plasma radius (0.01, 0.1, 1, 10) cm. The frequency interval $(0.01 - 6)*10^{15}$ Hz has been divided into 12 frequency groups. Two different ways of the averaging of absorption coefficients has been provided in our previous work [2] – Planck and Rosseland means. Comparison of the net emission (25) calculated using different mean values of absorption coefficient is presented in Fig. 1 for four different radius of the plasma cylinder.

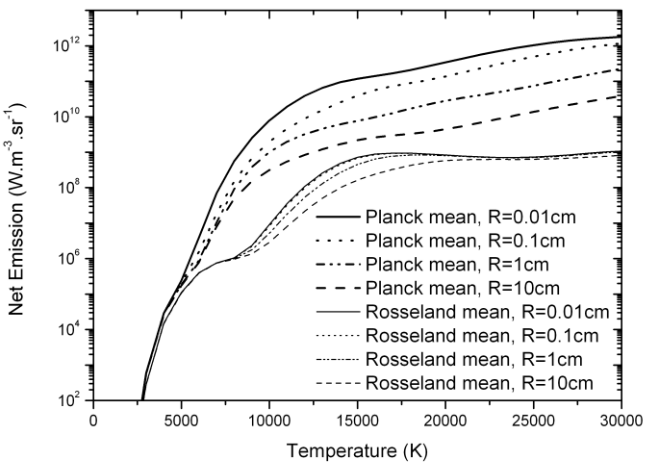
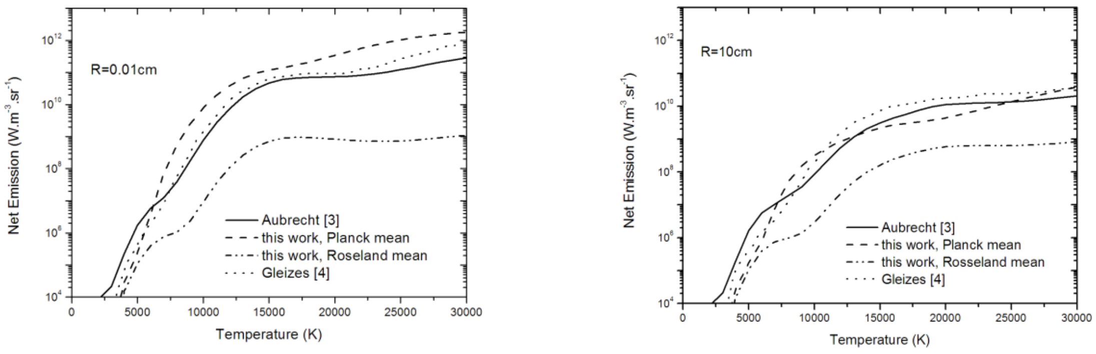


Fig. 1. Net emission of air isothermal plasma cylinder as a function of temperature for various thicknesses of the plasma and various mean values of absorption coefficients.

It can be seen that Rosseland averaging leads to lower values of the net emission which follows from the fact that Rosseland means underestimate the influence from the absorption peaks in the real absorption spectrum.

In Figs. 2 a), b) comparison is made of our calculations of net emission in air arc plasma of two radii with the results of Aubrecht [3] and Gleizes [4]. Planck averaging gives the results which are in satisfactory agreement with calculations of the other authors. Discrepancies between our results and those of Aubrecht and Gleizes can be explained by different approximate methods of calculation. Both Aubrecht and Gleizes use the method of the net emission coefficient with the integration over the real absorption spectrum.



a) b)

Fig. 2. Comparison of net emission of air calculated by various authors.

4. CONCLUSION

Calculations have been made of the net emission of radiation in the isothermal cylindrical air plasma for various radii of the plasma column. The multigroup P1-approximation has been used. For the Planck averaging general agreement with other sources of similar data has been reached. Discrepancies occur by reason of using various approximate methods of the solution of the equation of radiation transfer. Under assumption of isothermal plasma the equation (11) can be solved analytically, and the calculation of the net emission (25) is then very simple.

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